PROPAGATION OF A PLANE JET OF CONDUCTING LIQUID IN A MAGNETIC FIELD

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The paper examines the propagation of a plane free jet of incompressible liquid with variable conductivity in a transverse magnetic field at $Re_m \ll 1$.

In [1] the author examined the problem of propagation of a plane laminar jet—a source of incompressible conducting liquid in a nonuniform magnetic field. A self-similar solution was obtained for the case of constant conductivity of the medium. The present paper examines the same problem for a free jet allowing for variation of conductivity with temperature, under the simplest assumed dependence $\sigma = \sigma(T)$.

We consider a plane jet of conducting liquid discharging from a slit, the liquid having the same physical properties as the surrounding medium. We shall examine the case when the jet has considerable enthalpy, its temperature exceeding that of the surrounding medium: $T_m \gg T_\infty$. We shall assume that the jet propagates in a transverse magnetic field oriented along the y axis ($H_y = H_0 x^\eta$) with $Re_m \ll 1$.

We shall further assume that the dependence of conductivity on temperature is expressed by the power law

$$\sigma/\sigma_0 = (T/T_0)^m. \quad (1)$$

The initial system of equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu^2 H_y^2}{\rho}u, \quad (2)$$

$$\frac{\partial(ux^k)}{\partial x} + \frac{\partial(vx^k)}{\partial y} = 0,$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} + \frac{\sigma\mu^2 H_y^2}{\rho}u^2 \quad (3)$$

with boundary conditions

$$\frac{\partial u}{\partial y} = 0, \quad v = 0 \text{ when } y = 0, \quad (4)$$

$$u = 0 \text{ when } y = \pm\infty$$

under the power-law self-similarity transformations

$$u = u_m F'(\varphi), \quad T = T_m\Theta(\varphi), \quad u_m = ax^\alpha,$$

$$T_m = gx^\gamma, \quad \varphi = bx^\beta y \quad (5)$$

transforms to a system of ordinary differential equations

$$F''' + (3-2k)[(\alpha+1+2k)FF'' - 2\alpha F'^2] = N\Theta^m F', \quad (6)$$

$$\Theta'' + (3-2k)\Pr[(\alpha+1+2k)F\Theta' - 2\gamma F'\Theta] = 0, \quad (7)$$

with boundary conditions

$$F = 0, \quad F' = 1, \quad F'' = 0; \quad \Theta = 1, \quad \Theta' = 0 \text{ when } \varphi = 0;$$

$$F' = 0; \quad \Theta = 0 \text{ when } \varphi = \pm\infty. \quad (8)$$

It was assumed in (6) and (7) that

$$\beta = \frac{\alpha-1}{2}, \quad \eta = \beta - \frac{m\gamma}{2},$$

$$N = \frac{\sigma\mu^2 + H_0^2 g^m}{\nu b^2 T_0^m}, \quad \frac{a}{2\nu b^2} = 3-2k \quad (9)$$

(k = 0 applies to a plane jet, and k = 1 to a fan-shaped jet).

There is no term accounting for joule dissipation in the self-similar energy equation (7), because the last term on the right in the initial equation (3), as will be seen below, decreases with distance from the jet origin more quickly than the other terms.

By integrating (6) and (7) across the jet, we obtain formulas for the dependence of the self-similarity constants α, β, and γ on the magnetic interaction parameter N,

$$\alpha = -\frac{1}{3}\left(1 + 2k + \frac{N}{3}\frac{\int_0^\infty [\Theta(\varphi)]^m F'(\varphi)\,d\varphi}{\int_0^\infty [F'(\varphi)]^2\,d\varphi}\right),$$

$$\gamma = -\frac{\alpha+1+2k}{2}. \quad (10)$$

From the expressions for the momentum $\left(J_x = \int_{-\infty}^{\infty}\rho u^2 x^k dy \sim x^{\frac{3\alpha+1+2k}{2}}\right)$ and flow rate $\left(G = \int_{-\infty}^{\infty}\rho u x^k dy \sim x^{\frac{\alpha+1+2k}{2}}\right)$, bearing in mind that the latter may only increase along the jet axis, while the momentum is conserved in the ordinary discharge (N = 0), we find that the value of α for which the solution has physical meaning lies in the range $-1 - 2k < \alpha \le -1/3 - 2k/3$. In order to determine the dependence $\alpha = \alpha(N)$, as is seen from the expressions obtained (10), we need to solve (6) and (7). In a numerical solution we can ensure agreement of the solution with relations (10) by choosing a value of α (and it turns out to be unique) for which boundary conditions (8) are satisfied, with given values of the parameters m, Pr, and N. The results of integration on a EMU-type computer are presented in Figs. 1 and 2.

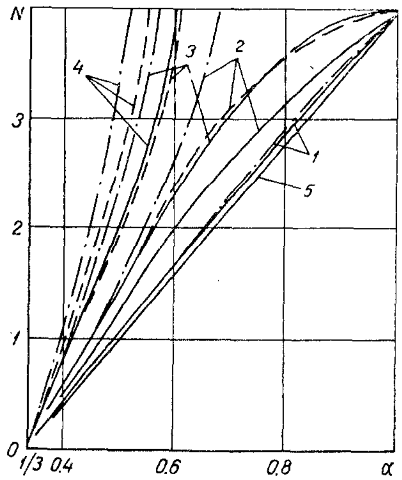

Fig. 1. Dependence of self-similarity constant α on the magnetic interaction parameter N for m = 1 (solid lines); 2 (dotted lines); 3 (dot-dash lines): 1) at Pr = 0.1; 2) 0.5; 3) 1; 4) 2; 5) at σ = const.

We shall briefly discuss the influence of each of the individual parameters. Increase of the magnetic interaction parameter, as may be seen from Fig. 1, leads to a more rapid drop in velocity (and in momentum) along the jet axis (the value of α increases; we recall that in the relation $u_m = \alpha x^{\alpha}$, the constant $\alpha < 0$). The dynamic thermal thicknesses of the jet then increase.

With increase of Pr number, the thickness of the thermal layer decreases (Fig. 2b), and the decelerating influence of the magnetic field is concentrated close to the jet axis. Therefore, for fixed values of parameters m and N there is less attenuation of the jet.

For Pr $\ll$ 0.1 and not overly large values of m, the relation between the values of α and N is nearly linear (Fig. 1), and is approximately determined by the relation, obtained for constant conductivity of the medium [1],

$$\alpha \cong -\frac{1}{3}(1 + 2k + N/2).$$

In this case the velocity profiles agree, within the limits of accuracy of the machine calculations, with the profiles for a purely hydrodynamic jet discharge ($F'(\varphi) = \mathrm{ch}^{-2}\varphi$). This is because the thermal layer thickness, for small Pr numbers, exceeds the dynamic boundary layer thickness. Therefore interaction of the conducting jet and the magnetic field occurs mainly in a weakly varying temperature field, i.e., with negligible variation of conductivity across the jet.

With increase of the parameter m in (1)—the dependence $\sigma = \sigma(T)$—there is a more rapid variation of conductivity across the jet, and the deceleration is largely localized in the region near the axis. Therefore as m increases, the effective thermal jet thickness (Fig. 2,c), as well as the value of α (Fig. 1), decrease. The width of the dynamic layer (in dimensionless coordinates) increases (Fig. 2,c).

The condition for nontrivial solution of the dynamic problem in self-similar transformations (5) is the integral quantity

$$\int_{-\infty}^{\infty} u^{\delta} x^{k} dy = S = \mathrm{const} > 0, \qquad (11)$$

which does not vary along the jet axis when $\delta = (\alpha - 1 - 2k)/2\alpha$. Then variation of δ in the range $1 < \delta \leq 2$ corresponds to variation of α in the range $-1 - 2k < \alpha \leq -1/3 - 2k/3$. The quantity S is assumed to be given. The invariant (11) is not unique for the self-similar transformations (5). For example, the integral $\int_{-\infty}^{\infty}\left(\frac{\partial u}{\partial y}\right)^{\frac{\alpha-1}{3\alpha-1}} dy$ is also conserved along the jet axis, and may be used to determine constants a and b. It is evident, however, that choice of different integral invariants leads only to different method of defining an arbitrary longitudinal velocity scale, and has no influence on its final magnitude.

Using the integral condition (11), in conjunction with the relation $a = 2(3 - 2k)\nu b^2$, we find the constants a and b:

$$a = \left[\frac{S}{\sqrt{(3-2k)2\nu}\int_{-\infty}^{\infty}[F'(\varphi)]^{\delta}\, d\varphi}\right]^{-2\alpha},$$

$$b = \frac{1}{\sqrt{2\nu(3-k)}}\left[\frac{S}{\rho\sqrt{2\nu(3-k)}\int_{-\infty}^{\infty}[F'(\varphi)]^{\delta}\, d\varphi}\right]^{-\alpha}. \qquad (12)$$

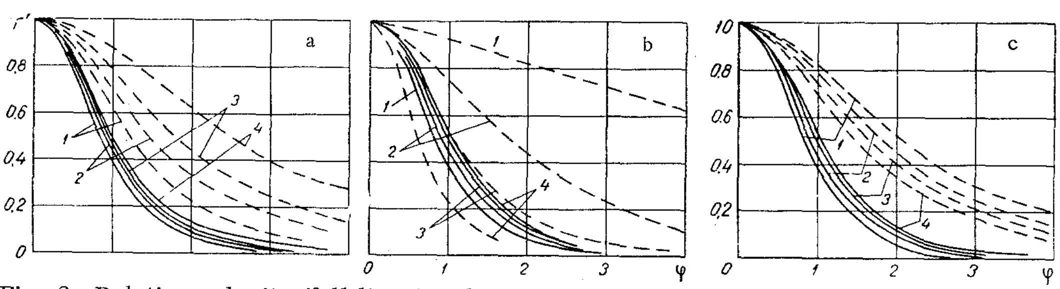

Fig. 2. Relative velocity (full lines) and temperature (dotted lines) profiles: a) with m = 1 and Pr = 0.5: 1) with N = 0, 2) 1, 3) 2, 4) 3; b) with m = 1 and N = 2: 1) with Pr = 0.1; 2) 0.5, 3) 1, 4) 2; c) with N = 2 and Pr = 0.5: 1) with m = 0, 2) 1, 3) 2, 4) 3.

From the condition of conservation of excess enthalpy

$$Q = \int_{-\infty}^{\infty} \rho C_p u x^k (T - T_\infty)\, dy = \rho C_p \int_{-\infty}^{\infty} u T x^k dy \qquad (13)$$

we determine the constant g:

$$g = \frac{b}{a} \frac{Q}{\rho C_p} \left[\int_{-\infty}^{\infty} F' \Theta \, d\varphi \right]^{-1}. \qquad (14)$$

In the special case examined below we succeeded in integrating the system (6), (7). We shall assume that the power-law relation, occurring in discharge of a jet of liquid with constant conductivity, between velocity and temperature profiles is conserved:

$$\Theta(\varphi) = [F'(\varphi)]^n \qquad (15)$$

(this relation will be obtained below from the thermal equation). Then (6) is rewritten in the form

$$F''' + (3 - 2k)[(\alpha + 1 + 2k) \times \times FF'' - 2\alpha F'^2] = NF'^{mn+1}. \qquad (16)$$

This equation is integrated in quadratures with m · n = = 0 (m = 0, the case of constant conductivity) and m · n = 1. In the latter case the solution will be the function

$$F'(\varphi) = \mathrm{ch}^{-2} \sqrt{1 - N/6}\, \varphi, \qquad (17)$$

where

$$\alpha = -\frac{1}{3}\left(1 + 2k + \frac{N}{3}\right). \qquad (18)$$

From the thermal equation (17) we determine an expression for dimensionless temperature profile

$$\Theta(\varphi) = [\mathrm{ch} \sqrt{1 - N/6}\, \varphi]^{-(3-2k)(\alpha+1+2k)\,\mathrm{Pr}} = [F'(\varphi)]^{\frac{(3-2k)(\alpha+1+2k)}{2}\mathrm{Pr}}, \qquad (19)$$

satisfying assumption (15), under which (6) was integrated. The condition m · n = 1 determines the relation between the quantities m, N, and Pr

$$\mathrm{Pr}(3 - 2k)(\alpha + 1 + 2k)\, m = 1.$$

The values of constants a, b, and g are determined as before by (12) and (13).

We note, finally, that some of the properties of self-similar flow of a jet of liquid of constant conductivity are conserved in the problem examined. This is related to the presence of a finite region where there is variation of the self-similarity constants and interaction between the basic parameters of the problem (momentum of the jet, excess enthalpy, and the parameters defining the external magnetic field).

NOTATION

u, v – longitudinal and transverse velocity components; x, y – longitudinal and transverse coordinates; T – temperature; H_y – magnetic field intensity; m – exponent in expression (1); a, b, and g – constants; S – integral invariant (given); G, J_x, Q – mass flow rate, momentum flux, and excess enthalpy of jet; H_0, T_0 – characteristic values of magnetic field intensity and temperature; T_m, T_∞ – values of temperature on jet axis and in surrounding medium, respectively; σ, σ_0 – conductivity, and its characteristic value; ν – kinematic viscosity; ρ – density; μ – permeability; k – numerical constant (k = 0, 1); F' – dimensionless velocity in a cross section of the jet; φ – reduced coordinate; Re_m – magnetic Reynolds number; N – magnetic interaction parameter; Θ – dimensionless temperature.

REFERENCE

1. K. E. Dzhaugashtin, IFZh [Journal of Engineering Physics], 8, no. 5, 1965.

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